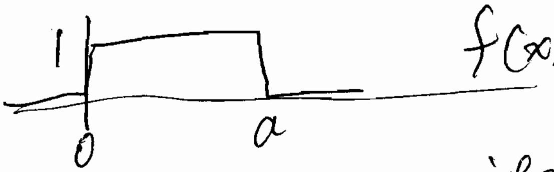

$f(x) = \theta(x) - \theta(x-a) \Rightarrow f(x-vt)$

e^{ik_0x}

$f(k) = \int_{-\infty}^{\infty} f(x)\, e^{ikx}\, dx$

$= \int_0^a e^{ikx}\, dx = \frac{1}{ik}\left(e^{ika}-1\right) = \frac{1}{i(k_0+k)}\left(e^{i(k_0+k)a}-1\right)$

$k=0 \Rightarrow \frac{ika}{ik} = a.$

$k \neq 0$ $\mathrm{Re}\left\{ -i\frac{1}{k}\left(e^{ika}-1\right) \right\} = \frac{\sin ka}{k}$ $\times \cos kx$

$\mathrm{Im}\left\{ \right\} = \frac{-\cos ka + 1}{k}$ $\times \sin kx$

$x - vt = x'$

$\int f(x-vt)\, e^{ikx}\, dx = \int_{-\infty} f(x')\, e^{ikx'}\, dx\, e^{ikvt}$

$f(x) = \int A(k) \cos(kx)\, dx + \int B(k) \sin(kx)\, dx$

$\int f(x) \cos(k'x)\, dx$

$= \int A(k) \int \cos(kx) \cos(k'x)\, dx\, dk$

$$f(x) = [\theta(x) - \theta(x-a)]\,\frac{e^{ik_0 x} + e^{-ik_0 x}}{2}$$

$$f'(k) = \int_{-\infty}^{\infty} f(x)\, e^{ikx}\, dx$$

$$= \int_0^a \frac{e^{i(k+k_0)x} + e^{i(k-k_0)x}}{2}\, dx$$

$$= \frac{e^{i(k+k_0)a} - 1}{2i(k+k_0)} + \frac{e^{i(k-k_0)a} - 1}{2i(k-k_0)}$$

$k = k_0$

$$f'(k) = \frac{e^{2ik_0 a} - 1}{4ik_0} + \frac{a}{2}$$

$k = k_0 - \Delta k \sim k_0 + \Delta k$

$$\mathrm{Re}[f'(k)] = \frac{+\sin(k+k_0)a}{2(k+k_0)} + \frac{+\sin(k-k_0)a}{2(k-k_0)}$$

$$\mathrm{Im}(f'(k)) = \frac{-\cos(k+k_0)a + 1}{2(k+k_0)} + \frac{-\cos(k-k_0)a + 1}{2(k-k_0)}$$